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Community of Heroes!

An Example of Nature's Mathematics: The Rainbow

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Introduction.

It is the author's contention that 'nature' is a wonderful resource and vehicle for teaching students at all levels about mathematics, be it qualitatively at elementary schools (shapes, circular arcs, polygonal patterns) or more quantitatively at middle and high schools (geometrical concepts, algebra, trigonometry and calculus of a single variable). This was the motivation for writing *A Mathematical Nature Walk* (as well as the somewhat more advanced *Mathematics in Nature*). Within the realm of nature the subject of *meteorological optics* is a particularly fascinating one; it includes the study of the rainbow as well as others such as ice crystal halos and glories. Obviously there is some physics involved in the explanation of these phenomena, but fortunately it is not necessary to go into a lot of physical detail in order to appreciate the value of geometry, trigonometry and high-school calculus concepts used in modeling the beautiful rainbow arcs in the sky.

For students in elementary school there is a variety of angle-based concepts that can be addressed when discussing rainbows. Thus, 'solar altitude' is the angle the *direction* to the sun makes with the horizontal, just as the direction to the top of a tree makes an angle with the direction of its shadow on the ground (in fact that angle is exactly the solar altitude!). By making a large paper cone to mimic the 'rainbow cone' and varying the angle at which students hold it, (Figure 3), they will see that if the sun is very low (i.e. close to the horizon), then the rainbow arc is almost a complete semicircle, whereas if the sun is too high (altitude greater than 42^{o}), then the top of the rainbow is below the horizon and therefore not visible (unless the observer is on a hill or in flight; see http://www.slate.com/content/dam/slate/blogs/bad_astronomy/2014/09/01/circular_rainbow.jpg.CROP.original-original.jpg). If the student (or anyone!) is fortunate enough to see a nearly semicircular rainbow, then the angle between the two 'ends' of the rainbow and the observer – its 'angular diameter' – is twice 42^{o}, which is not far from a right angle!

What about middle-school students? In the summer of 2015 I was privileged to teach a dozen specially selected 6th – 8th grade students in the Virginia STEAM Academy at Old Dominion University. The acronym refers to Science, Technology, Engineering and Applied Mathematics. The topics covered included rainbows, ice crystal halos, water waves, glitter paths and sunbeams; additionally, the topics ‘Guesstimation’ (i.e. back-of-the-envelope problems that require estimation) and ‘dimensional analysis’ (i.e. what happens as things get bigger?) were incorporated into the week-long class. Given that the mathematical background of these students included algebra, geometry and trigonometry, much of the material discussed in this article was covered, and the *results* from the calculus-based topics were presented qualitatively (and very successfully) by engaging the students on their understanding of maxima and minima, and applying those ideas in this context.

Doing the mathematics.

The *primary* rainbow is caused by light from the sun entering the observer's eye after it has undergone one reflection and two refractions in myriads of raindrops. An additional internal reflection produces a frequently-observed *secondary* bow, and so forth (but tertiary and higher bows are rarely, if ever, seen with the naked eye for reasons discussed below). By adding all the contributions to angular deviations of the ray from its original direction, the middle- or high-school student can verify that for a primary bow the ray undergoes a total deviation of $D(i)$ radians, where

$$D(i) = 2(i - r) + (\pi - 2r) = \pi + 2i - 4r(i), \quad (1)$$

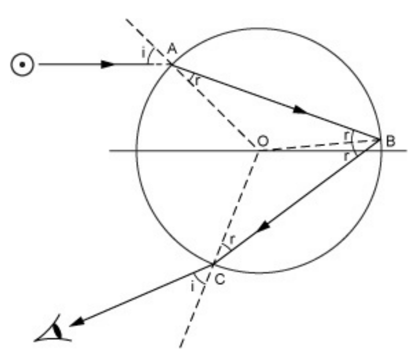

Figure 1. The path of a ray inside a spherical raindrop which, along with myriads of other such drops, contributes to the formation of a primary rainbow ($k = 1$). The deviation angle $D(i)$ referred to in the text (see equation (1)) is the obtuse angle between the extension of the horizontal ray from the sun and the extension of the ray entering the observer’s eye. Its value is approximately 138°. Its supplement, 42°, is the semi-angle of the ‘rainbow cone’ in Figure 3.

in terms of the angles of incidence (i) and reflection (r) respectively (see Figure 1 where the incident ray is refracted and reflected inside the spherical drop; Figure 2 illustrates the ray path for the secondary bow). *But what is this angle*? Essentially it is the direction through which an incoming ray from the sun is ‘bent’ by its interaction with the drop to reach the observer’s eye (the reader is referred to the caption for Figure 1 for more details).

The angle of refraction inside the drop is a function of the angle of incidence of the incoming ray. This relationship is being expressed in terms of Snell's famous law of refraction, namely $\sin i = n \sin r$, where n is the *relative index of refraction* (of water, in this case). This relative index is defined as the ratio of the speed of light in medium I (air) to the speed of light in medium II (water); note that $n > 1$; in fact $n \approx 4/3$ for the rainbow, but it does depend slightly on wavelength (this is the phenomenon of dispersion, and without it we would only have bright ‘whitebows’!). The article by Austin & Dunning (1991) provides a helpful summary of the ‘calculus of rainbows,’ as does the even briefer ‘Applied Project’ in Chapter 4 of Stewart (1998).

In view of Snell’s law the high school student should attempt to write the angle of refraction r in terms of the angle of incidence i using the inverse sine function, thus:

$$r = \arcsin\left(\frac{\sin i}{n}\right). \quad (2)$$

Hence equation (1) may be rewritten as

$$D(i) = \pi + 2i - 4\arcsin\left(\frac{\sin i}{n}\right). \quad (3)$$

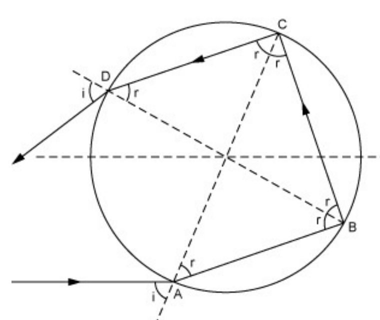

Figure 2. The corresponding ray path for the secondary rainbow, arising because of a second reflection within the raindrops.

By examining the graph of $D(i)$ in Figure 4 it is seen that for $i \in [0, \pi/2]$ (which is the only interval of interest for physical reasons), the condition for an extremum (minimum in this case) implies there exists a critical angle of incidence i_c such that $D'(i_c) = 0$. To prove this the student may either use implicit differentiation of equation (1) (with subsequent use of Snell's law) to obtain

$$\frac{dr}{di} = \frac{\cos i}{n\cos r}, \quad (4)$$

or directly differentiate the expression (3) and equate it to zero to find the critical angle i_c from the resulting expression below, i.e.

$$\frac{1}{4} = \frac{\cos^2 i}{n^2 - 1 + \cos^2 i}, \quad (5)$$

from which it can be found that

$$i \equiv i_c = \arccos\left(\frac{n^2-1}{3}\right)^{1/2}. \quad (6)$$

Thus, with a generic value for n of 4/3, $i_c \approx 1.04$ radians, or about 59.4° for the primary bow.

As noted above this extremum is a *minimum*, i.e. $D''(i_c) > 0$, as can be shown by differentiating the expression (1) a second time. In fact, by noting from equation (1) that $D''(i) = -4r''(i)$ and utilizing equation (4) it follows that (after some algebraic manipulation)

$$\frac{d^2r}{di^2} = -\frac{\left(n^2-1\right)}{n^3}\frac{\sin i}{\cos^3 r} < 0, \qquad (7)$$

So, $D''(i_c) > 0$ as indicated. Note that in this instance it was not necessary to specify i_c so the result is a global one, i.e. the concavity of the graph of $D(i)$ does not change in $[0, \pi / 2]$, the interval of physical interest.

Exercise for the student: Using equations (3) and (6) show that the minimum angle of deviation (the 'rainbow angle') is

$$D(i_c) = \pi + 2\arccos\left(\frac{n^2-1}{3}\right)^{1/2} - 4\arcsin\left(\frac{4-n^2}{3n^2}\right)^{1/2}. \quad (8)$$

Each internal reflection adds an amount of $\pi - 2r$ radians to the total deviation of the incident ray. Thus, for k internal reflections within a raindrop, a term $k(\pi - 2r)$ is added to the angle through which an incident ray is deviated, (see Figure 2, for the secondary bow, $k = 2$), yielding the expression

$$D(i) = 2(i-r) + k(\pi - 2r) = k\pi + 2i - 2(k+1)r(i)$$
$$= \; k\pi + 2i - 2(k+1)\arcsin\left(\frac{\sin i}{n}\right). \quad (9)$$

Note that the result in equation (9) is modulo 2π. Although realistically $k \leq 2$ (see below for details), with k internal reflections the corresponding result for the critical angle of incidence that gives rise to the minimum deviation is

$$i_c = \arccos\left(\frac{n^2-1}{k(k+2)}\right)^{1/2}. \quad (10)$$

This result is established using exactly the same method to arrive at equation (6). For the primary bow ($k = 1$) this reduces (as it should) to equation (6) above. Additionally, equation (9) reduces to equation (3) for $k = 1$ since $k(k + 2) = 3$.

It is an interesting trigonometric exercise to eliminate all dependence on the angle of incidence (as Kepler did in 1652) to prove from equation (8) that

$$D\left(i_c\right) = 2\arccos\left[\frac{1}{n^2}\left(\frac{4-n^2}{3}\right)^{3/2}\right]. \quad (11)$$

To achieve this, rewrite equation (8) as

$$\frac{D(i_c)-\pi}{2} = \arccos\left(\frac{n^2-1}{3}\right)^{1/2} - 2\arcsin\left(\frac{4-n^2}{3n^2}\right)^{1/2}$$

$$\equiv A - 2B, \quad (12).$$

Then, by expanding the equation

$$\sin\left(\frac{D(i_c)-\pi}{2}\right) = \sin(A-2B),$$

it is possible to write $\cos[D(i_c)]$ in terms of $\sin A$, $\cos A$, $\sin B$ and $\cos B$, each of which can be found easily from the definitions of A and B in equation (12). The result is a rather nasty expression which can be reduced algebraically to equation (11). Voilà! This has been generalized to higher-order bows (see Adam 2008), but it would take us too far afield to describe here; essentially the same ideas are involved.

Some numerical values.

Thus far, we have been describing a generic, colorless type of rainbow. For a 'generic' monochromatic rainbow (the 'whitebow' referred to above), the choice $n = 4/3$ yields, from expression (11),

$$D(i_c) = D\left(\arccos\left[\frac{9}{16}\left(\frac{20}{27}\right)^{3/2}\right]\right) \approx 138^\circ. \qquad (13)$$

The supplement of this angle (≈ 42°) is the semi-angle of the rainbow 'cone' formed with apex at the observer's eye, the axis being along the line joining the sun to the eye, extended to the antisolar point (see Figure 3).

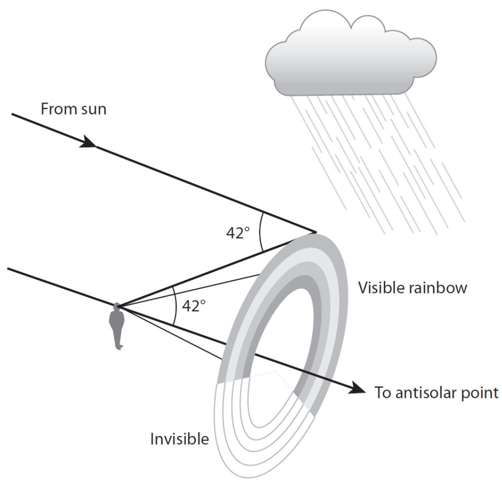

Figure 3. The 'rainbow cone' for the primary rainbow. For the secondary bow (k = 2) the cone semi-angle is approximately 51°, as may be calculated from equations (9) and (10).

So what happened to the colors of the rainbow? They have of course been there all along, and all we need to do is to utilize the fact that the refractive index n is *slightly* different for each wavelength of light. Blue and violet light get refracted more than red light; the actual amount depends on the index of refraction of the raindrop, and the calculations thereof vary a little in the literature, because the wavelengths chosen for red and violet may differ slightly. Thus, for red light with a wavelength of 656 nm (1 nm = 10^{-9} m), the cone semi-angle is about 42.3°, whereas for violet light of 405 nm wavelength, the cone semi-angle is about 40.6° an angular spread of about 1.7° for the primary bow. (This is more than *three times* the angular width of a full moon!) The corresponding values of the refractive index differ very slightly: $n \approx 1.3318$ for the red light and $n \approx 1.3435$ for the violet – less than a one percent increase! Similar (though slightly wider) dispersion occurs for the secondary bow, but the additional reflection reverses the sequence of colors, so the red color in *this* bow is on the inside edge of the arc. In principle more than two internal reflections may take place inside each raindrop, so higher-order rainbows, i.e. tertiary (k = 3), quaternary, (k = 4) etc., are possible. Each additional reflection of course is accompanied by a loss of light intensity because of transmission out of the drop at that point, so on these grounds alone, it would be expected that even the tertiary rainbow (k = 3) would be difficult to observe or photograph without sophisticated equipment; however recently several orders beyond the secondary have been identified and photographed (see the cited articles by Edens, and Edens & Können). The reader's attention is also drawn to the superb website on atmospheric optics, in particular the following link: http://www.atoptics.co.uk/rainbows/ord34.htm.

It is possible to derive the angular size of such a rainbow after any given number of reflections using equations (9) and (10) (Newton was the first to do this). Newton's contemporary, Edmund Halley, noted that the third rainbow arc should appear as a circle of angular radius nearly 40° around the sun itself. The fact that the sky back-

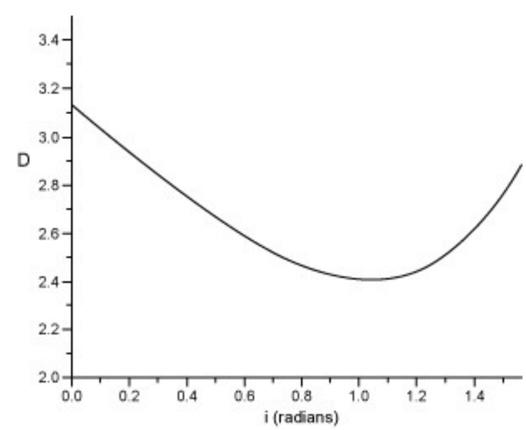

Figure 4. The graph of the deviation angle $D(i)$ for the primary bow from equation (3) as a function of the angle of incidence (both in radians). Note that the minimum deviation of approximately 2.4 radians (≈ 138°) occurs where the critical angle of incidence $i_c \approx 1.04$ radians (≈ 59.4°). These values may be calculated directly using equations (3) and (6). The graph shows that above and below i_c there are rays deviated by the same amount (via the horizontal line test), indicating that at i_c these two rays coalesce to produce the region of high intensity we call the rainbow.

ground is so bright in this vicinity, coupled with the intrinsic faintness of the bow itself, would make such a bow almost, if not, impossible to see or find without sophisticated optical equipment. **Exercise for the student:** Use the generic value for the refractive index of water, $n = 4/3$, in equations (9) and (10) to show for the tertiary rainbow ($k = 3$) that $i_c \approx 70.6^o$ and $D(i_c) \approx 321^o$, so the 'bow' is at about an angle of 39° from the incident light direction. In fact, this will appear behind the observer as a ring around the sun!

Exercise for the student: Calculate the angular width subtended at the eye by a 'baby aspirin' held at arm's length. Then see if you can 'cover' the full moon by extending your arm while holding the aspirin!

An experiment: "road-bows."

Have you ever noticed a rainbow-like reflection *from a road sign* when you walk or drive by it during the day? Tiny, highly reflective spheres are used in road signs, sometimes mixed in paint, or sometimes sprayed on the sign. Occasionally, after a new sign has been erected, quantities of such 'microspheres' can be found on the road near the sign (see http://apod.nasa.gov/apod/ap040913.html for an excellent picture of such a bow). I have had my attention drawn to such a find by an observant student! It is possible to get samples of these tiny spheres directly from the manufacturers, and reproduce some of the reflective phenomena associated with them. In particular, for glass spheres with refractive index $n \approx 1.51$ scattered uniformly over a dark matte plane surface, a small bright penlight provides the opportunity to see a beautiful near-circular bow with an angular radius of about 22° (almost half that of an atmospheric rainbow). In such an experiment this bow appears to be suspended above the plane as a result of the stereoscopic effects because the observer's eyes are so close (relatively) to the spheres compared with passing several yards from a road sign. More details of the mathematics can be found in the article by Crawford (1998) and Chapter 20 of Adam (2012).

Related topics in meteorological optics.

Note that in the list of topics below each meteorological phenomenon can be examined as a topic in mathematical physics because the subject of optics is very mathematical. At times, it required very sophisticated mathematics. The author recommends another enrichment activity in which students search for each of the topics (and others) below on the 'Atmospheric Optics' website mentioned above: http://atoptics.co.uk/. There is a vast selection of topics (with *many* photographs) to choose from, including shadows, ice crystal halos around the sun or moon, 'sundogs,' reflections, mirages, coronas, glories, sun pillars as well as, of course, rainbows. The advantage of this site (and its 'sister' site, Optics Picture of the Day (OPOD: http://atoptics.co.uk/opod.htm)) is that students at all levels, elementary, middle and high school, will be able to find material of interest to them. These sites are replete with straightforward physical explanations and illustrations of the phenomena, but there is little, if any, mathematical discussion so they can be appreciated in a scientifically accurate way by students at any level of mathematical proficiency. The book *A Mathematical Nature Walk*, together with the more advanced *Mathematics in Nature* cited in the bibliography, can provide a starting point for both teachers and students interested in pursuing some of the mathematical aspects of these phenomena. As a further example, a very brief description of glories (with an associated 'student teaser') is provided below.

Although *ice-crystal halos* are only briefly mentioned in the preceding paragraph, students at all levels can be encouraged to look for them. These can appear around the sun or full moon with surprising frequency (though it must be emphasized again that you should never look directly at the sun; block it off with your hand or a convenient chimney!). They are formed by sunlight passing through myriads randomly oriented, nearly regular hexagonal prismatic ice crystals, composing cirrus clouds, the very highest type of cloud we generally see (during the day at least). In distinction to rainbows, the most common halos are smaller in angular radius (about 22° as opposed to 42°) and exhibit a reddish tinge on the inside of the arc – a reversal of colors compared with the primary bow. This is because, unlike the mechanism producing the primary bow, there is no reflection occurring within the crystals to produce these particular halos, only refraction. I live about a mile from Old Dominion University and walk to my office; as a result I generally see such halos (and other types also) several times a month; sadly, far more frequently than I witness rainbows. On an otherwise clear night, a full moon embedded in a thin cirrus cloud may exhibit similar such halos, which can be quite prominent by virtue of the moon being so much less bright than the sun. Indeed, I have frequently been contacted by friends and students who witness the latter but have never noticed a halo around the sun!

Exercise for the student: Imagine a regular hexagonal prism with a light ray entering side '1', and exiting side '3' (see the *Atmospheric Optics* website for more details and its interactive 'mouse' tasks to discover the minimum angle of deviation for both rainbows and halos). Using the same geometric, trigonometric and calculus concepts applied to rainbows in the body of the article, show that the minimum angle of deviation for such rays is about 22°, the angular radius of the most commonly visible halo.

Student teaser: When I lived in England I saw many more rainbows than I do living in Norfolk, Virginia. Why do you think this was? (No, it was not because the annual rainfall where I lived was more than it is in Norfolk – in fact it's rather less!). Think about *latitude*: I lived at about 52°N; now I live at about 37°N, fifteen degrees further south. (You can imagine how excited I was to see the constellation of Orion and Sirius (the 'Dog star') so much higher in the winter night sky than when I lived in England!)

Glories.

Mountaineers and hill climbers have noticed on occasion that when they stand with their backs to the low-lying sun and look into a thick mist below them, they may see a set of colored concentric circular rings (or arcs thereof) surrounding the shadow of their heads. Although an individual may see the shadow of a companion, the observer will see the rings only around his or her head. They may also be seen (if you know where to look) from airplanes. This is the meteorological optics phenomenon known as a glory. Cloud droplets essentially 'backscatter' sunlight back towards the observer in a mechanism similar in part to that for the rainbow. The glory, it is sometimes claimed, is formed as a result of a ray of light tangentially incident on a spherical raindrop being refracted into the drop, reflected from the back surface and reemerging from the drop in an exactly antiparallel direction (i.e. 180°) into the eye of the observer, but this is actually incorrect (see the 'student teaser' below).

Student teaser: Why is the ray path allegedly associated with the formation of a 'glory' as illustrated in Figure 5 (and in some meteorology textbooks) *incorrect*? Use equation (3) to investigate this.

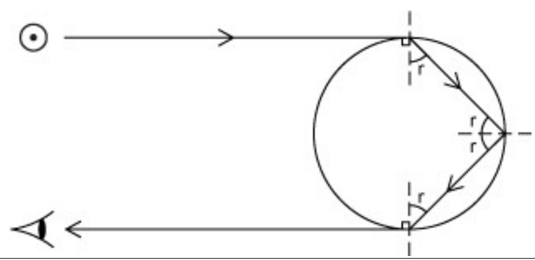

Figure 5. An incorrect ray path explanation for the glory.

Conclusion.

This article presents some of the basic mathematical concepts and techniques undergirding a relatively common (and beautiful) phenomenon in meteorological optics. The analysis presented here does not contain new mathematics; it can be found from many sources because the subject of meteorological optics has been around for a very long time! What is emphasized, however, is the presentation of these ideas as a potential enrichment topic for (i) 'qualitative' mathematical modeling in elementary classrooms and (ii) more quantitative approaches in middle and high school classrooms. It should also be noted that the many more subtle features associated with these and other optical effects in the atmosphere require far more powerful and sophisticated mathematical tools to explain them. Nevertheless (though space does not permit it), aspects of the above-mentioned phenomena of ice crystal halos and glories may also be discussed at the level presented here. More details may be found in the references listed. It is hoped that this article will also 'whet' the appetite of interested instructors and students to pursue these aspects in more detail.

A further suggestion may be made. The website *Earth Science Picture of the Day* (EPOD: epod.usra.edu), which is a service of the Universities Space Research Association (USRA), publishes photographs from a variety of subject areas: geology, oceanography, space physics, meteorological optics, agriculture, and many more. Anyone is invited to submit their photograph of an interesting optical or geological phenomenon, and is encouraged to write a short summary for the layman explaining the picture and, where possible, the basic science behind it. A recent submission by the author (August 15th, 2016), for example, uses simple proportion to estimate the height of a tree canopy using the 'pinhole' elliptical patches of light cast on the ground by gaps in the leaves of the tree (http://epod.usra.edu/blog/2016/08/estimating-tree-height-using-natural-pinhole-cameras.html). For a propos that is the topic of this article, see http://epod.usra.edu/blog/2017/07/streaky-rainbow-in-zion-national-park.html. The site provides useful educational links for the daily pictures and is a valuable resource for teachers and students alike.

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Appendix: More mathematical patterns in nature.

What follows below is obviously only a partial list of patterns that the attentive observer might see on a "nature walk," and could form the basis of enrichment activities at all levels of student exposure to mathematics. The elementary, middle or high school teacher could adapt the material for his or her own students. Here are some possible topics:

Basic two dimensional geometric shapes that occur (approximately) in nature can be identified:

- *Waves on the surfaces of ponds or puddles expand as circles;*
- *Ice crystal halos commonly visible around the sun are generally circular;*
- *Rainbows have the shape of circular arcs* (*as noted already*);
- *Tree growth rings are almost circular.*

But there are many other obviously non-circular and non-planar patterns:

- *Hexagons: snowflakes generally possess hexagonal symmetry;*
- *Pinecones, sunflowers and daisies (amongst other flora) have spiral patterns associated with the well-known Fibonacci sequence;*
- *Ponds, puddles and lakes give scenes of approximate reflection symmetry (depending on the position of the observer);*
- *Cross-sections of various fruits also exhibit interesting symmetries;*
- *Spider webs have polygonal, radial and spiral-like features;*
- *Long bendy grass has an approximately parabolic shape;*
- *Starfish (suitably arranged) exhibit pentagonal symmetry;*

- *The raindrops that scatter "rainbow" light into the eye of an observer essentially lie on cones with vertices at the eye (as discussed in this article);*
- *Cloud patterns, mud cracks and also cracks on tree bark can exhibit polygonal patterns;*
- *Clouds can also form wavelike "billow" structures with well-defined wavelengths, just as with ripples that form around rocks in a swiftly flowing stream;*
- *In three dimensions, snail shells and many seashells and curled-up leaves are helical in shape and tree trunks are approximately cylindrical.*

In view of these patterns, even at an elementary level, many pedagogic mathematical investigations can be developed to describe such patterns - for example estimation, measurement, geometry, functions, algebra, trigonometry and calculus of a single variable. Basic examples might include:

- *The use of similar triangles and simple proportion;*
- *A table of tangents to estimate the height of trees;*
- *Measuring inaccessible horizontal distances using congruent triangles.*

Simple proportion can again be used in estimation problems, such as:

- *Finding the number of blades of grass in a certain area, or the number of leaves on a tree.*

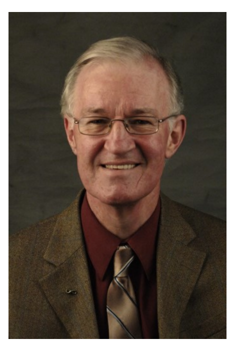
More geometric ideas appear when studying topics such as:

- *The relationship between the branching of some plants, such as sneezewort (Achillea ptarmica), and the Fibonacci sequence can be investigated;*
- *The related "golden angle" can be studied, and its occurrence on many plants (such as laurel) investigated;*
- *The angles subtended by the fist, and the outstretched hand, at arm's length can be estimated and used to identify the location of "sundogs" (parhelia) and ice crystal halos on days with cirrus clouds near the sun.*

Consequences of "the problem of scale" and geometric similarity can also be investigated. This applies in particular to the size of land animals; the relationship of surface area to volume, and its implications for the relative strength of animals. By considering (and constructing) cubes of various sizes, much insight can be gained about basic biomechanics in the animal kingdom, and much fun (and learning!) may be had by thinking about such questions as:

- *Why King Kong could not really exist, and*
- *Why elephants are not just large mice.*

Furthermore, simple ideas such as scale enable us to compare, at an elementary level, metabolism and other biological features (such as strength) in connection with pygmy shrews, hummingbirds, beetles, flies and other bugs, ants and African elephants to name a few groups!



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