Supplementary Appendix

1.1. Directly Estimating Turnout Bias with a Demonstration from the 2000 Presidential Election

In this paper I develop an algebraic method of estimating turnout bias, which I define as the residual category that arises once I subtract distributional and apportionment bias from total bias. The strength of this method is that it is easy to calculate and does not require state turnout or population data.[1] One potential critique of the algebraic method of estimating turnout bias is that these estimates also include residual sources of bias introduced by the presence of third party candidates or the interactions between the various sources of bias. In this appendix, I assess the degree to which residual sources of bias shape my algebraic estimates of turnout bias. I do this by developing a direct estimate of turnout bias and then assessing how this estimate compares to the algebraically derived alternative. I find that the two methods produce broadly similar results.

I directly measure turnout bias by comparing the difference in the intercepts from two sets of logged odds equations run on two sets of seat-vote curves, one centered a 50:50 distribution of the mean state-level vote share (the estimate of just distributional bias) and the second centered a 50:50 distribution of the mean state-level population-weighted vote share (which is effectively distributional bias plus turnout bias). Turnout bias arises when there is a difference between a party's

[1] This method also has the advantage of sidestepping the question of how to measure turnout rates. The difficulty with measuring turnout is determining what the denominator of the turnout (e.g. percent of CVAP over 18 that cast a ballot vs. the percent of eligible voters that cast a ballot vs. the percent of registered voters that cast a ballot). This method effectively accounts for any potential registration biases by treating turnout bias as a residual category.

mean state-population weighted vote share and the party's share of the national two-party vote. Because a party's average state-population weighted vote share does not take turnout into account (or in other words, assumes turnout is equal in every state), a party's proportion of the average state-population weighted vote share will be higher than the party's national two-party vote share if the party's base of electoral support is concentrated in states with low turnout.

I work through how to do these calculations using the 2000 presidential election as an example. Turnout bias arises from one party winning states with lower average turnout compared to the turnout in states won by the opposing party. In 2000, Republican George W. Bush narrowly lost the two-party popular vote to Democratic Al Gore 49.5 to 50.5 despite winning the Electoral College 271 to 266. The disjunction between the popular vote and Electoral College outcome indicates that the Electoral College was biased in favor of the Republican Party. As I noted in the main body of the text, there is evidence that distributional bias (an inefficient distribution of electoral support) cut against the Republican Party in 2000, which suggests that apportionment and/or turnout bias worked in the Republicans favor.

A comparison of the mean Republican state-level two-party vote shares and the Republican national two-party vote helps to illustrate why this is the case. As I noted in the preceding paragraph, Republican George W. Bush won 49.5 of the national two-party vote. However, Bush's mean two-party vote share was 51.9 (the median was 50), almost two and a half points higher than this national two-party vote share. This number implies that on average Bush won states by a larger margin than did his Democratic opponent Al Gore, a fact that is confirmed by figure A1.

Figure A1 displays the vote margin in each state during the 2000 and 2004 presidential elections (positive numbers reflect pro-Republican vote margins). The notable feature of this figure is the left-skewed distribution of vote margins. Democrat Al Gore won three states (and DC) by a margin of 20 points or greater in 2000. Comparatively, George W. Bush won 11 states by more than 20 points, meaning the Bush's distribution of votes was less efficient than Gore's, who typically won states by a smaller margin than his opponent. This inefficient distribution of votes led to distributional bias in favor of the Democrats.

I investigate this possibility in more detail in columns one and two in table A1. Column two contains an estimate of total bias using Tufte's (1973) logged odds method applied to a hypothetical seats-votes curve centered on an even 50:50 split of the national two-party vote. Column three contains an estimate of distributional bias, again using Tufte's logged odds method applied to a hypothetical seats-votes curve, the difference being this hypothetical distribution is centered at a mean state-level two-party vote share of 50 percent. The first equation estimates there was a 3.37-point total bias in favor of the Republican Party; however, the second equation estimates there was a 9.6-point distributional bias in favor of the Democrats. The discrepancy between estimates of total and distributional bias indicate that other forms of bias (apportionment, turnout, and potentially from the presence of Ralph Nader, a third party candidate), were working in favor of the Republican candidate.

(Figure A1 Here)

Malapportionment is an obvious source of pro-Republican bias in the 2000 election. Republican George W. Bush won 30 of the 51 (counting the District of

Columbia) states despite failing to capture a plurality of the popular vote. Bush performed especially well in the states with small populations, which are systematically overrepresented in the Electoral College. This is evidenced by the candidates' respective electoral vote totals if I subtract the two electoral votes each state receives regardless of their population. Removing these two votes helps to bring a states number of electoral votes closer in line with its population. Removing these two votes from each state would shift the candidates total Electoral vote totals from 271/266 to 211/224, reversing the outcome of the election.

I analyze the degree of apportionment bias in greater depth with a third analysis of a hypothetical seats-votes curve. The difference between this analysis and the previous analysis of total bias is that I remove two electoral votes from each state's total (the seats-votes curve is still centered on an even distribution of the national two-party vote) in an effort to remove the main source of apportionment bias from the analysis.[2] Removing the electoral votes associated with each state's senate seats reveals the presence of apportionment bias. There is an estimated 1.15-point pro-Republican bias once the major source of apportionment bias is removed, compared to the estimated 3.37-point overall bias in favor of the Republicans. This discrepancy means that there is 2.22-point (3.37-1.15) apportionment bias in favor of the Republican Party, which is ultimately rooted in the Republican Party's strong performance in the sparsely populated Great Plains and Mountain West states.

[2] Like I mentioned in the main text, this procedure removes the major source of apportionment bias, but not all potential sources of apportionment bias. This second set of apportionment bias stems from the integer problem that exists in the apportionment process.

However, there is still a considerable amount of unexplained pro-Republican bias even after I account for the main source of malapportionment. This finding suggests that additional sources of bias are present. The most obvious additional source of bias stems from differences turnout levels across constituencies. Partisan bias would arise from George W. Bush winning states with lower average turnout than the Democrat Al Gore. Figure A2, which plots state-level turnout among the voting eligible population (VEP) against the Republican share of the two-party vote, lends support to this possibility (evidenced by the cluster of states located in the bottom right quadrant of the figure). [3] On average, turnout in states won by Bush was considerably lower turnout compared to the states won by Gore, at 53.7 percent versus 58.4 percent respectively. The discrepancy between the average turnout rate in Democratic and Republican states opens up the possibility that turnout bias was a critical factor in explaining the overall pro-Republican bias in the 2000 election.

(Figure A2 Here)

I investigate this possibility with a fourth and final logged odds analysis of a hypothetical seats-votes curve. The results of this analysis are reported in column five of table 1A. The seats-votes curve is centered at a mean population-weighted state-level two-party vote share of 50 percent. Centering the distribution at the population weighted state-level two-party vote share assumes that each state contributed to the overall average at a rate commensurate with the state's population. Stated differently, this analysis assumes that turnout rates are equal

[3] Turnout data was obtained from Michael P. McDonald's United States Elections Project: http://www.electproject.org/2000g

across jurisdictions. The average VEP-weighted two-party Republican vote share is 49.8, which is lower than the unweighted average of 51.9 but still higher than the national Republican two-party vote share of 49.5. This estimate can then be directly compared to the estimate of distributional bias alone and the difference between the two estimates (if any) is directly attributable to the presence of turnout bias.[4]

(Table A1 Here)

The analysis in column four in confirms that turnout bias played a significant role in shaping total bias. The analysis of distributional bias alone revealed the presence of a significant distributional 9.7-point bias in favor of the Democrats. The joint estimate of distributional and turnout bias reveals a net bias of .75 in favor of the Democratic Party. This estimate is statistically indistinguishable from zero and implies an 8.95-point pro-Republican turnout bias. The interpretation of this finding is that in the 2000 presidential election, the partisan effects of distributional and turnout bias effectively canceled one another out. Figures A1 and A2 illustrate why this was the case. The Republican Party won states by a larger average margin than the Democrats; however, the states the Republican Party won had lower turnout than the states carried by the Democrats. The analysis presented in the main text of the paper suggests that the partisan direction of distributional and turnout bias have historically cut in opposite directions. This in depth analysis of the 2000 presidential election illustrates exactly why this is the case.

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[4] In this analysis, the primary source of apportionment bias, the two electoral votes that correspond with each state's Senate seats, have been removed from the total number of electoral votes. This is done in an effort to isolate only distributional and turnout bias.

So what proportion of the residual bias I derive algebraically in the main text is explained by the presence of turnout bias? My preceding analysis of the 2000 election revealed there is a 1.9-point gap between the summed effect of these three estimates of distributional, apportionment, and turnout bias and the overall estimate of pro-Republican bias. This unexplained difference might be evidence a fourth and unaccounted for source of bias arising from third party votes or simply a product of the uncertainty associated with my measurement procedure. The core takeaway here is that the vast majority of my algebraically derived estimate of residual bias is explained by the presence of turnout bias. Other sources of bias make up a very small proportion of the residual bias category.

Overall, the evidence that I have presented here indicates that distributional, apportionment, and turnout bias were all factors in 2000. On the balance, these partisan biases favored the Republican Party.

1.2 Comparing My Estimates with Those of Johnston and Pattie (2014)

As readers that are versed in the literature on bias and responsiveness as well as the literature on the Electoral College will note, my estimates of total bias and its constituent sources differ from those of Pattie and Johnston (2014), who developed their own estimates of partisan bias in the article, '"The Electors Shall Meet in their Respective States:" Bias and the U.S. Presidential Electoral College, 1960-2012,' which appeared in *Political Geography* in 2014. Some of the most notable differences between my estimates and the author's estimates occur in 2000, where the author's argue the Republicans benefitted from a substantial amount of efficiency bias (what I term distributional bias). In fact, the authors argue that efficiency bias provided the Republicans a 50 plus seat advantage in the Electoral College (Johnston and Pattie 2014, page 43, figure 8). The authors also argue that apportionment bias worked in favor of the Republicans to the degree of a 15-seat advantage and there was no discernable amount of turnout bias (page 42, figure 6).

Pattie and Johnston's estimates conflict with the estimates I provided in the main text of the paper and the detailed analysis of the 2000 election in the previous section. This raises the question of which set of estimates is correct? I laid out the rationale for both my estimation strategy and the specific estimates of bias in the 2000 election. In this section I lay out simple, but I argue compelling, reasoning as to why Johnston and Pattie's estimates of the components of bias are implausible.

The reason for doubt is simple. In 2000, Johnston and Pattie project a 50 plus seat Republican advantage as a result of efficiency/distributional bias in addition to the presence of a smaller but still significant pro-Republican apportionment bias and no discernable turnout bias. However, in 2004, according to the authors' estimates, the patterns of bias in the Electoral College fundamentally change. The authors estimate that in 2004, there was a massive shift in efficiency bias in favor of the Democrats. The authors estimate over a 100 electoral vote shift in efficiency bias (page 43, figure 8), which is close to a 20 percent swing of the *total* seats in the Electoral College. Such a shift is implausible. This kind of shift in the partisan bias in the Electoral College would require a considerable shift in the distribution of the parties' bases of electoral support. The problem with this is that there was hardly any substantive shift in the parties' bases of support from 2000 to

2004, let alone a major one. The results of the 2004 election are extremely similar to the results of the 2000 election.

Figure A3 contains three scatter plots. The first is the linear fit between the state level Republican vote margins 2000 and 2004. The second panel is the linear fit between the Republican two-party vote in 1996 and 2000. The third is the linear fit between the Republican two-party vote in 1996 and 2004. In addition, table A2 contains three regressions, each assessing the bivariate relationship between the state-level two-party vote outcomes in the three aforementioned pairs of elections. The most notable feature of the relationship between all three of these pairs of elections is the tight linear relationship between the state-level vote outcomes. As the regression model in Table A2 demonstrates, regressing the 2000 Republican two-party vote shares on the 2004 Republican vote shares produces a coefficient of one—there is literally a *one to one* relationship between the state-level vote outcomes. In addition, the R-squared of this bivariate relationship is .95, which indicates a tight linear fit (evidenced by the plot) with very little noise. Bush did roughly 1.5 points better across the board, with no systematic shift in the parties' state-level bases of electoral support. The relationship between the 1996 and 2000 election outcomes is also fundamentally similar, with a coefficient of 1.13 and a R-squared of .915. Even the relationship between the 1996 and 2004 election returns are highly stable, with a coefficient of 1.08 and a R-squared of .897.

(Figure A3 and Table A2 Here)

The point of showing these figures and regression models is to illustrate that there was a remarkable level of consistency in state level election returns between

1996 and 2004. The idea that a constant cannot explain a change is a simple logical precept. State-level Republican vote shares remained virtually constant between 2000 and 2004, and highly stable between 1996 and 2004.[5] Yet, Johnston and Pattie estimate that there were massive shifts in the patterns of total and efficiency/distributional bias from 1996 to 2004, with both total and efficiency/distributional bias running in favor of the Democrats in 1996, then heavily in favor of the Republicans in 2000, before dramatically shifting once again back in favor of the Democrats in 2004. [6]

There is no way that these fundamentally similar election returns could produce such fundamentally different patterns of distributional bias. The authors' estimates are simply not plausible given the continuity in the parties' geographic base of support that characterizes the elections in question.

1.3 Dealing with the Presence of Third Party Candidates

One of the weaknesses of Tufte's logged odds methods of estimating partisan bias and swing ratio is that it is difficult to account for the presence of third party

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[5] Note that my estimates of total bias and its constituent sources are stable throughout this time period, a finding that lends credibility to the validity of my estimation method.

[6] Moreover, Johnston and Pattie estimate that turnout bias played a negligible role in the 2000 election. This finding stands in stark contrast to both the data displayed in figure A2 that suggests important differences in turnout in states won by Bush versus those won by Gore. The importance of these turnout differentials is confirmed by the logged odds analysis presented in column four in table A1. What is even more troubling is that Johnston and Pattie estimate that there was a significant turnout bias in favor of the Republicans in 2004. This finding is in spite of the fact that the turnout gap between Republican and Democratic states shrank between 2000 and 2004 (4.7 versus 4.2). Thus, it not only seems that turnout bias had a large relatively sizeable effect during both the 2000 and 2004 elections, but the magnitude of this bias should decrease between 2000 and 2004, not increase (this pattern is in line with my estimates of turnout bias presented in the main text).

candidacies. Third party candidates generally do not exert a great deal of influence in most American elections, although there have certainly been exceptions to this general statement. The exceptions to this general trend come in two forms. The first are instances where a third party candidate won a large proportion of the vote (e.g. 1992, 1968, etc.). The second are the instances where a third party candidate won a small but ultimately pivotal number of votes in a key state (such as Nader did in Florida in the 2000 election), which had the effect of tipping the state and ultimately the entire election. The question here is how I can incorporate the effects of these third party candidacies into my estimates of bias and responsiveness? The answer is that it is relatively easy to account for the presence of small third parties but more difficult to utilize the logged odds method when third parties win a substantial proportion of the vote.

Dealing with the presence of small third parties, such the Green Party (led by Ralph Nader) in 2000 is relatively straightforward, although it requires altering the setup of the logged odds models. The logged odds models reported in the paper all rely on the Republican share of the two-party vote as the empirical cornerstone. I assume that the Republican Party wins a state's electoral votes when its share of the two-party vote exceeds 50 percent. Obviously, the two-party vote explicitly excludes the votes obtained by all third parties. Therefore, in order to account for the presence of these third party candidacies it is necessary to move from using the two-party vote to using a party's share of the overall vote. This also requires shifting the standard of winning from "did the Republican Party win a majority of the two-party vote?" to "did the Republican Party win a plurality of the vote?" When calculating the hypothetical seats-votes curved, the third party's share of the vote is fixed, and the Republican and Democratic shares are allowed to vary. Other than that, the procedure remains largely the same.

The challenge with applying the logged odds method to elections that involve three or more relevant parties (and what has thus far limited the applicability of the logged odds method in many European elections) relates to the how to calculate the hypothetical seats-votes curve. The 2015 British election illustrates this challenge quite well. Four parties won more than 5 percent of the vote in the 2015 UK general election and a fifth, the Scottish National Party, won 4.7 percent of the vote, although its vote was geographically concentrated almost entirely in Scotland. The problem here is how to calculate how many seats a particular party will win across a range of national-level vote shares. If the third party candidacy is strong enough, this will require shifting the centering point of the distribution away from the 50 percent mark to some lower threshold. This requirement introduces some challenges. For instance, if I were to vary Labour's share of the vote from 20 to 40 percent (Labour's actual vote share was just over 30 percent) should I assume that Labour gains or loses support exclusively to/from the Conservatives or is it better to assume that Labour gains or loses support equally from/to all other parties? There is not an immediately clear answer. Estimating the relationship between changes in a party's share of the vote and its corresponding share of the seats becomes more complicated when there are more than two parties.

One potential solution to this problem is through the use of simulations. Right now the primary problem with applying the logged odds method to multiparty elections is there is no method to determine degree to which a party loses or gains support from all other parties when constructing a hypothetical seats-votes curve. Does a party gain or lose support from/to all other parties equally or predominately from one rival party. A simulation-based approach based on thousands of automated iterations is one method to address this problem (a version of this approach, the Judgeit program, has already been developed by Gelman and King and is available as an R package). Each simulation would allow the tradeoffs between a party's gain (loss) in support and the corresponding loss (gain) in other parties' support to vary (ranging from pulling support entirely from one party to pulling from all other parties equally). One could then assess the central tendency of the distribution of simulated seats-votes curves to determine the degree of partisan bias and responsiveness.

This method is more computationally intensive than Tufte's basic logged odds analysis, but it can incorporate estimates of swing and bias derived from wide range of scenarios. Incorporating these wide range of scenarios is necessary given the large number of potential tradeoffs when calculating vote swings with more two relevant parties.

Table A1: Four Logged Odds Estimates of Bias and Responsiveness

	Total Bias	*Distributional Bias*	*Removing Apportionment Bias*	*Distributional +Turnout Bias*
Swing	**5.03**	**4.84**	**5.15**	**5.01**
	(0.13)	(0.19)	(0.14)	(0.13)
Alpha	**0.15**	**-0.39**	.046	-.03
	(0.03)	(0.05)	(.033)	(.030)
N	21	21	21	21
R^2	0.988	0.970	0.985	0.988
Total Bias	3.37	-9.6	1.15	-.75

Table Note: Statistically Significant Entries (p<. 05) in Bold

Table A2: Bivariate OLS Models Estimating the Relationship Between State-Level Vote Outcomes

	2000	*2000*	*2004*
Republican Two-Party Vote 2004	**1.00**		
	(0.033)		
Republican Two-Party Vote 1996		**1.13**	**1.08**
		(0.049)	(0.052)
Constant	-0.013	-0.0026	0.028
	(0.018)	(0.023)	(0.025)
Observations	51	51	51
R-squared	0.950	0.915	0.897

Table Note: Statistically Significant Entries (p<. 05) in Bold

Table A3: Total Bias and Swing Ratio Estimates for the U.S. Electoral College 1872-2012 while Removing the Major Source of Apportionment Bias

	1872	*1876*	*1880*	*1884*	*1888*	*1892*	*1896*	*1900*	*1904*	*1908*	*1912*	*1916*	*1920*	*1924*	*1928*	*1932*	*1936*	*1940*
Swing	**5.07**	**5.87**	**5.16**	**6.97**	**7.23**	**6.06**	**3.87**	**5.16**	**2.77**	**4.67**	**4.42**	**5.87**	**3.04**	**2.28**	**5.45**	**4.29**	**6.96**	**6.39**
	(0.32)	(0.29)	(0.29)	(0.35)	(0.46)	(0.30)	(0.12)	(0.45)	(0.076)	(0.31)	(0.32)	(0.70)	(0.35)	(0.19)	(0.22)	(0.24)	(0.61)	(0.56)
Alpha	**-0.14**	**-0.14**	**-0.31**	**-0.21**	**-0.24**	**-0.21**	**0.15**	-0.15	**-0.095**	**-0.15**	0.0010	-0.23	-0.040	0.0025	**-0.30**	**-0.16**	**-0.54**	**-0.39**
	(0.08)	(0.07)	(0.07)	(0.08)	(0.11)	(0.07)	(0.03)	(0.11)	(0.02)	(0.08)	(0.08)	(0.17)	(0.09)	(0.05)	(0.05)	(0.06)	(0.15)	(0.14)
N	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21
R^2	0.93	0.96	0.94	0.96	0.93	0.96	0.98	0.87	0.99	0.92	0.91	0.79	0.80	0.89	0.7	0.94	0.87	0.87

	1944	*1948*	*1952*	*1956*	*1960*	*1964*	*1968*	*1972*	*1976*	*1980*	*1984*	*1988*	*1992*	*1996*	*2000*	*2004*	*2008*	*2012*
Swing	**7.04**	**7.42**	**7.86**	**9.14**	**8.75**	**6.64**	**8.21**	**7.04**	**10.1**	**8.78**	**9.83**	**10.1**	**9.59**	**6.83**	**5.15**	**5.07**	**4.65**	**4.01**
	(0.54)	(0.56)	(0.34)	(0.48)	(0.40)	(0.32)	(0.23)	(0.36)	(0.37)	(0.26)	(0.30)	(0.44)	(0.56)	(0.33)	(0.14)	(0.23)	(0.49)	(0.18)
Alpha	**-0.48**	**-0.53**	**-0.29**	**-0.47**	**-0.22**	0.12	0.074	**0.33**	**-0.16**	-0.0033	0.13	0.044	0.15	-0.040	0.046	0.037	-0.0076	-0.034
	(0.13)	(0.14)	(0.08)	(0.12)	(0.10)	(0.08)	(0.06)	(0.09)	(0.09)	(0.06)	(0.07)	(0.11)	(0.14)	(0.08)	(0.03)	(0.06)	(0.12)	(0.05)
N	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21
R^2	0.90	0.90	0.97	0.95	0.96	0.96	0.99	0.95	0.97	0.98	0.98	0.97	0.94	0.96	0.99	0.96	0.82	0.96

Table Note: Statistically Significant Entries ($p<.05$) in Bold

Figure A1: the Distribution of Vote Margins in the 2000 and 2004 Presidential Elections

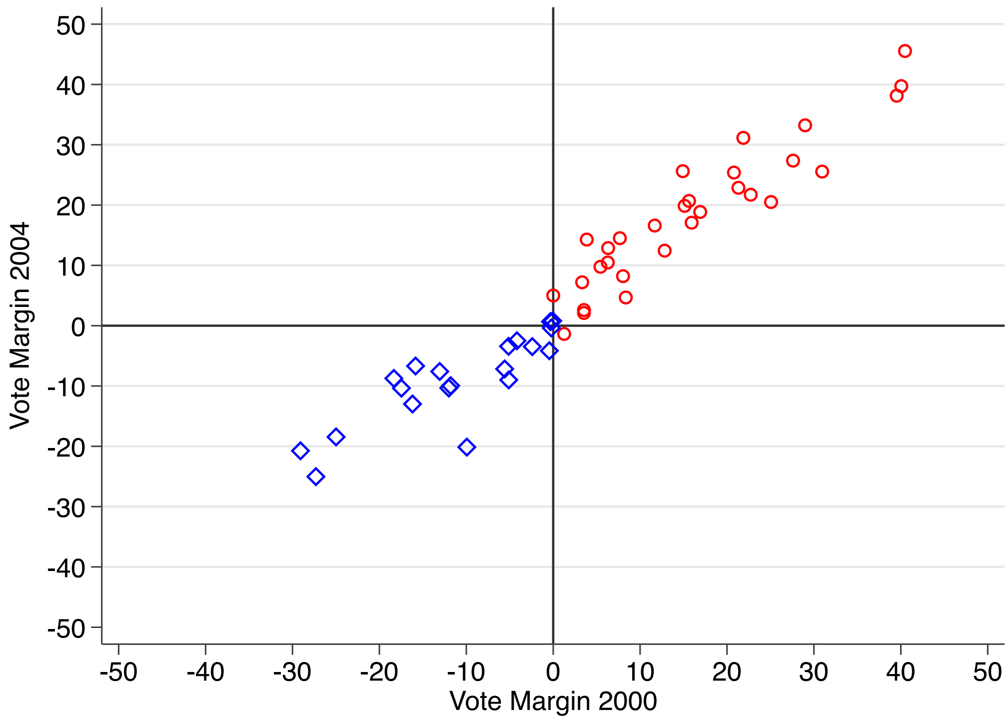

Figure Note: States won by Gore are colored blue and states won by Bush are colored red. One notable outlier, the District of Columbia, has been omitted to enhance the presentation.

Figure A2: the Distribution of Turnout Rates Across States by Republican Two-Party Vote Share in the 2000 Election

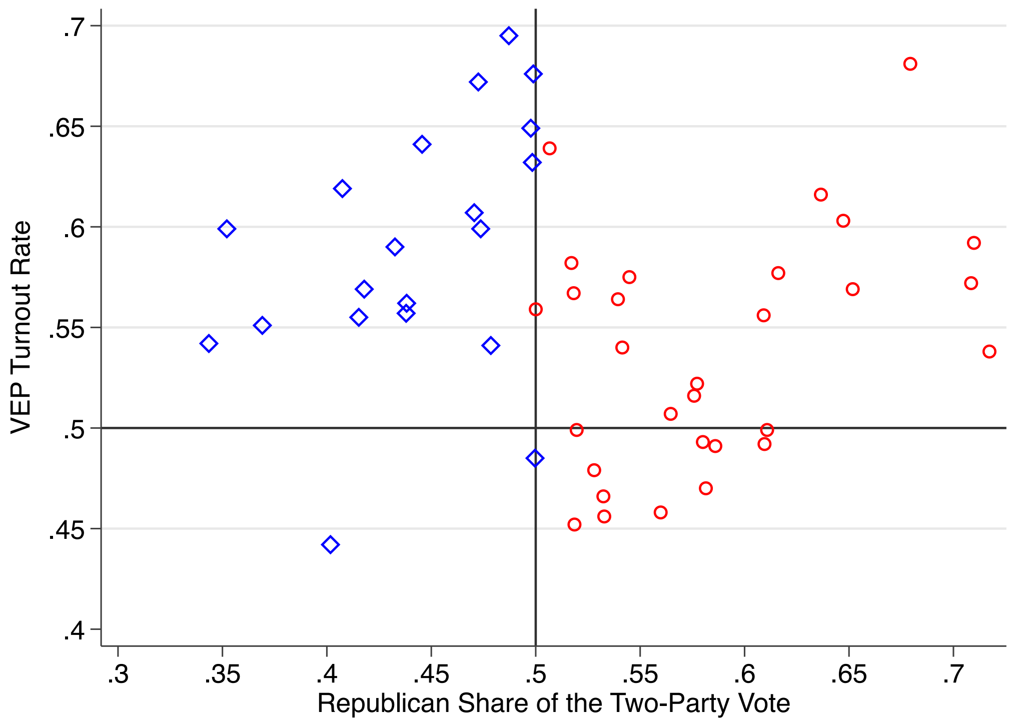

Figure Note: States won by Gore are colored blue and states won by Bush are colored red. One notable outlier, the District of Columbia, has been omitted to enhance the presentation.

Figure A3: The Relationship Between State Two-Party Vote Shares, 2000-2004, 1996-2000, and 1996-2004

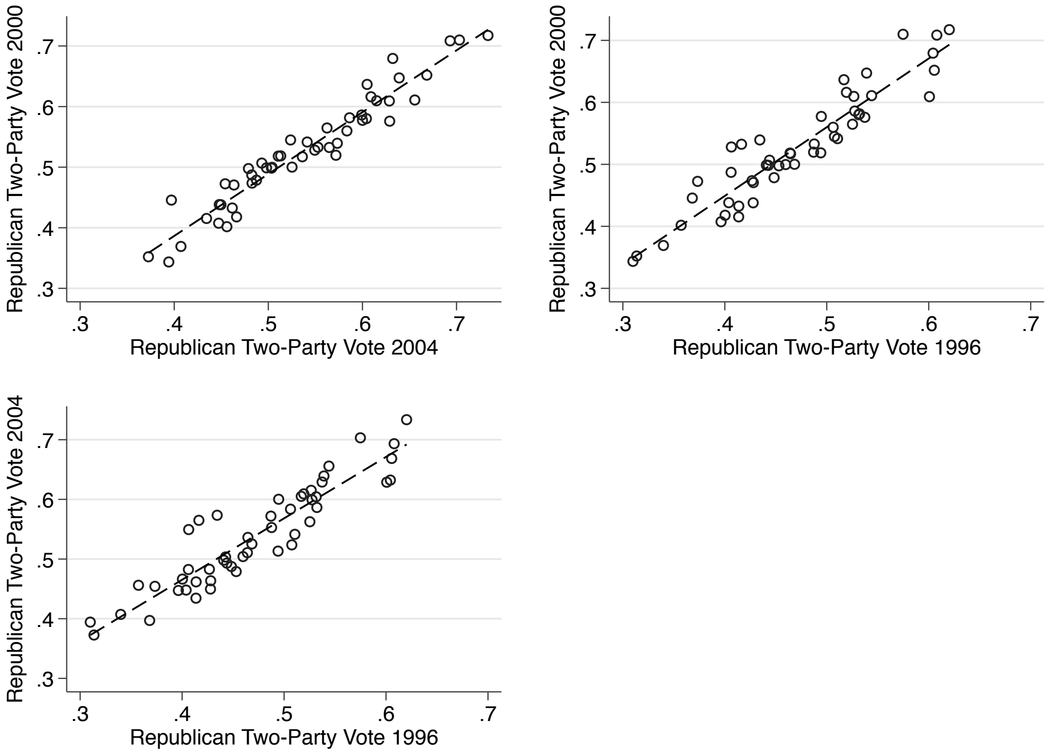

Figure Note: One notable outlier, the District of Columbia, has been omitted to enhance the presentation.